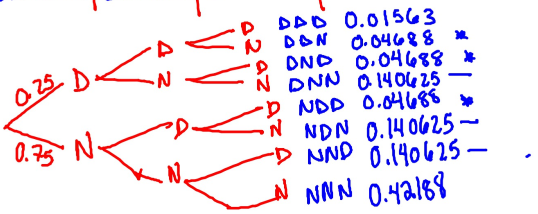
Example: Computer chips have a 25% chance of being defective. Create the probability distribution for the # of defective chips in a sample of 3.

D	0	1	2	3
P(D)	0.42188	0.42188	0.14064	0.01563



CH. 17 PROBABILITY MODELS

Binomial models:
* Interested in the number of successes in a set # of trials
* 4 conditions that must apply:
  - Only 2 possible outcomes (success/failure) correct incorrect
  - Probability of success remains constant (called ***p***) 0.25
  - Number of trials is set/known (called ***n***) $n=10$
  - Independent trials
    * **10% condition:** If we cannot assume independence we can proceed as long as the sample is smaller than 10% of the population (pop ≥ 10n)

* If these 4 conditions apply, we have a **Bernoulli trial**

Binomial Variable

Notation:

B(n, p) $N(\mu, \sigma)$

Ex:

$\mu_X = n*p$ ← # successes

$\mu_c = 10*0.25 =$ 2.5 questions

$\sigma_X = \sqrt{n*p*(1-p)}$

$\sqrt{n \cdot p \cdot q}$ (p: success, q: failure)

$\sigma_c = \sqrt{10*0.25*0.75}$

$\sigma_c = 1.369$ questions

Binomial probabilities:

Example (from earlier):
Computer chips have a 25% chance of being defective. Create the probability distribution for the # of defective chips in a sample of 3.
(a) What is the probability of having 2 or more defective chips?
(b) What is the probability of having 1 or less defective chips?
(c) What is the probability of having exactly 2 defective chips?

STEP 1: Bernoulli? check to see....

1. success = defective, failure = not defective
2. $p = 0.25$
3. $n = 3$
4. pop ≥ 10n There are more than 30 computer chips ⇒ independence

B(3, 0.25)

STEP 2: Create probability distribution (we did this before)

X	0	1	2	3
P(X)	0.42188	0.42188	0.14063	0.01563

STEP 3: ANSWER THE QUESTIONS (notation!!) $n=20$

(a) What is the probability of having 2 or more defective chips?
$P(X \geq 2) = 0.15626$

(b) What is the probability of having 1 or less defective chips?
$P(X \leq 1) = 0.84376$

(c) What is the probability of having exactly 2 defective chips?
$P(X=2) = 0.14063$

D	P(D)
0	
1	
2	
⋮	
20	

QUICKER WAY TO GET PROBABILITIES:
Formula:

$P(X = k) = ({}_nC_k)(p^k)(q^{n-k})$ ← combinations

Same example: B(3, 0.25)

X	0	1	2	3
P(X)	0.421875	0.421875	0.140625	0.015625

$P(X=0) = (3\text{nCr}0)(0.25\wedge 0)(0.75\wedge 3)$

$P(X=1) = (3\text{nCr}1)(0.25\wedge 1)(0.75\wedge 2)$

Example: I am playing a game in which I have only a 39% chance of winning. I am playing 4 times. Create the probability distribution below: $P(X=k) = ({}_nC_k)(p^k)(q^{n-k})$ (Binom) $B(4, 0.39)$

X (k)	P(X) P(X=k)
0	(4 nCr 0)(0.39^0)(0.61^4) = 0.13846
1	(4 nCr 1)(0.39^1)(0.61^3) = 0.35409
2	(4 nCr 2)(0.39^2)(0.61^2) = 0.33958
3	⋮ = 0.14474
4	⋮ = 0.02313

1

x	P(X)
0	0.13846
1	0.35409
2	0.33958
3	0.14474
4	0.02313

B(4, 0.39)

P(X=0)

binompdf(4, 0.39, 0)

So let's answer some easy questions:

P(X=2) =

P(X<2) =

P(X≥3) =

P(2≤X≤4) =

Now let's change the sample size to ~~10~~. ~~50~~ 100 B(10, 0.39)

x	P(X)
0	.
1	.
2	.
3	.
4	.
5	
6	
7	
8	
9	
10	

P(X=9) =

P(X<4)=

P(X≥6)=

P(5≤X≤7)=

P(20≤X≤50)

Would you want to answer these questions for a sample size of 50? Of 100? NO! **So we can use the calculator!**

For $\boxed{P(X=k)}$

- Use ... binompdf(n, p, k)
- k = the number you are looking for...
  Example: P(X = 5)........ k = 5
- pdf = probability distribution function
  (gives each individual outcome's probability)

For P(X ≤ k)

- Use ... binom**cdf**(n, p, k)
- k = the number you are looking for...
  Example: P(X ≤ 6)........ k = 6
- Notice that is ONLY GIVES YOU: LESS THAN OR EQUAL TO
- cdf = cumulative distribution function... adds up all the probabilities below

Ex: P(X ≤ 3) = P(X=0) + P(X=1) +P(X=2) + P(X=3)

$P(X>7) = 1-P(X\le 7)$

Example: John is taking archery. He has a 30% chance of hitting the target each time he shoots. He shoots 8 times. B(8, 0.30)

1) What is the probability that he hits the target 4 times?
   P(X=4) = binompdf(8, 0.30, 4) = 0.1361
2) What is the probability that he hits the target 2 times or less?
   P(X ≤ 2) = binomcdf(8, 0.30, 2) = 0.5518
3) What is the probability that he hits the target at least 3 times?
   P(X ≥ 3) = 1 − P(X ≤ 2) = 1 − binomcdf(8, 0.3, 2) = 0.4482
4) What is the probability that he hits the target less than 5 times?
   P(X < 5) = P(X ≤ 4) = binomcdf(8, 0.3, 4) = 0.942
5) What is the probability that he hits the target more than 6 times?
   P(X > 6) = 1 − P(X ≤ 6) = 0.0013
6) How many times do we expect him to hit the target? (average!)
   $\mu_x = n \cdot p = (8)(0.3) = 2.4$ hits
7) What is the standard deviation of the number of times he hits the target?
   $\sigma_x = \sqrt{8 * 0.3 * 0.7} = 1.296$ hits

Example: John is t *ANSWERS* as a 30% chance of hitting the target each time he shoot s. B(8, 0.30)

1) What is the probability that he hits the target 4 times?
$P(X = 4)$ = binompdf(8, 0.30, 4) =

2) What is the probability that he hits the target 2 times or less?
$P(X \leq 2)$ = binomcdf(8, 0.30, 2) =

3) What is the probability that he hits the target at least 3 times?
$P(X \geq 3) = 1-P(X \leq 2)$ = 1-binomcdf(8, 0.30, 2) =

4) What is the probability that he hits the target less than 5 times?
$P(X < 5) = P(X \leq 4)$ = binomcdf(8, 0.30, 4) =

5) What is the probability that he hits the target more than 6 times?
$P(X > 6) = P(X \geq 7) = 1-P(X \leq 6)$ 1-binomcdf(8, 0.30, 6) =

6) How many times do we expect him to hit the target? (average!)
$E(X) = \mu_X = 8*0.30 = 2.4$ times

7) What is the standard deviation of the number of times he hits the target?
σ_X = sqrt(8*0.30*0.70) = 1.296 times

Try this example on your own: 150 businesses are sent mailings asking them to answer a survey question and send the mailing back. The probability of nonresponse is 55%. $p=0.45$

1) What is the average number of businesses that WILL respond?
2) What is the std. deviation of the number that will respond?
3) What is the probability that 75 businesses will respond?
4) What is the probability that 60 businesses or less will respond?
5) What is the probability that 60 businesses or more will respond?
6) What is the probability that less than 60 businesses will respond?
7) What is the probability that greater than 60 businesses will respond?

$\mu_X = 90 = n(0.45)$

8) What number of surveys would you have to send out if you wanted to be able to expect to get 90 back?
9) What is the probability that between 50 and 70 businesses will respond?

$P(50 < X < 70)$

$P(51 \leq \boxed{X \leq 69}) \leftarrow$

$P(X \leq 69) - P(X \leq 50)$

51

69

0

1) μ_X = 67.5 businesses
2) σ_X = 6.093 businesses
3) $P(X = 75) = 0.0306$
4) $P(X \leq 60) = 0.1251$
5) $P(X \geq 60) = 1-P(X \leq 59) = 0.9058$
6) $P(X < 60) = P(X \leq 59) = 0.0942$
7) $P(X > 60) = 1-P(X \leq 60) = 0.8749$
8) n = 200

B(150, 0.45)

9) $P(50 < X < 70) = P(51 \leq X \leq 69)$

$= P(X \leq 69) - P(X \leq 50)$

$= 0.6271$

Complete worksheet #1

#1 -- 8

Worksheet answers:

1) (a) Yes
(b) Yes
(c) Only if you replace between picks, and if they give number of trials in the problem
(d) No- more than 2 outcomes
(e) No- more than 2 outcomes
(f) No- probability of success isn't constant (different chance of effectiveness for each person)
(g) Yes
(h) Yes
(i) No- more than 2 outcomes
(j) Yes

2) Binomial?

- succes = alarm failing   failure = alarm working
- p = 0.05 and is constant
- Independent trials stated
- n = 6

B(6, 0.05)

(a) P(X = 3) = binompdf(6, 0.05, 3) = 0.002

(b) P(X < 2) = P(X ≤ 1) = binomcdf(6, 0.05, 1) = 0.9672

(c) P(X = 0) = binompdf(6, 0.05, 0) = 0.7351

③ B(10, 0.25) $P(X \geq 6) = 1 - P(X \leq 5) = 0.0197$

④ B(15, 0.90) $P(X \geq 9) = 1 - P(X \leq 8) = 0.9997$

⑤ B(10, 0.60) $P(X = 3) = 0.0425$

⑥ B(9, 0.60) ⓐ $P(X \geq 5) = 1 - P(X \leq 4) = 0.7334$

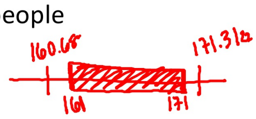
ⓑ $P(X = 7) = 0.1612$

ⓒ $P(X > 3) = 1 - P(X \leq 3) = 0.9006$

7) $\mu = 200 * 0.83 = 166$ people

$\sigma = \sqrt{200 * 0.83 * 0.17} = 5.3122$ people

8) $\mu \pm \sigma = (160.6878, 171.3122)$



DISCRETE = can only take whole number values

WITHIN the range = $(161 \leq \boxed{X \leq 171)}$

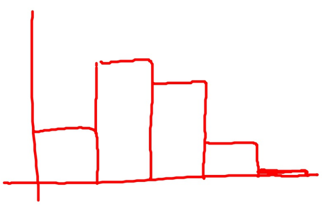
$P(161 \leq X \leq 171) = P(X \leq 171) - P(X \leq 160) = 0.6998$

Complete the experiment in the notes

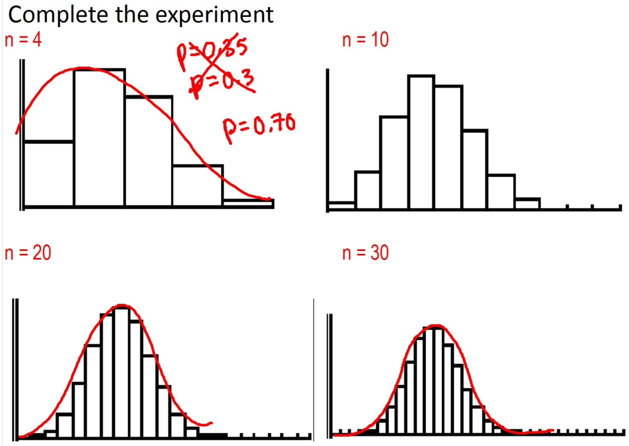
(If you dont have the notes, let me know- I have extra copies of the expt)

B(4, 0.35)

x	P(x)
0	0.17851
1	0.38448
2	0.31054
3	0.11148
4	0.01501



Complete the experiment



Ch. 17: Probability Models:

Binomial Random Variables: LARGE SAMPLE SIZE

What happens to the shape of the Binomial Random Variable when *n* is large?

when n is large → shape is normal

What is considered a "large enough" n (for the shape to look normal)??

Check:

$n \cdot p$, $n \cdot q$ ≥ 10 → normal

So if the check passes...

- We can say that the distribution is approx. normal, and can use normalcdf!
- Calculator:

  normalcdf(lower bound, upper bound, mean, std. dev)
- Same mean and std dev. that we learned before:

  $\mu_X = n*p$

  $\sigma_X = \sqrt{n*p*q}$

Example 1:

It is said that 75% of people pay their credit card bill on time. If we take a sample of 125 adults, what is the chance that over 80 of them paid their bill on time this past month?

Bernoulli? ✓

$B(125, 0.75)$

Check:

$(125)(0.75)$, $(125)(0.25)$ ≥ 10 ✓

Work:

* $N(93.75, 4.841)$ $P(X>80) = \text{normalcdf}(80, E99, 93.75, 4.841)$

$= 0.9977$

Example 2:

Assume 15% of adults jog regularly. We survey 1,000 adults what is the probability that between 120 and 160 people in our sample jogs?

$B(1000, 0.15)$

check:

$1000(0.15)$, $1000(0.85)$ ≥ 10 ✓

$N(150, 11.292)$

$P(120 \leq X \leq 160)$

$\text{normalcdf}(120, 160, 150, 11.292)$

$= 0.8081$

Complete worksheet 2

#1 -- 4, 10

Answers:

1)	(a) yes	(b) no	(c) no
2)	(a) n = 100	(b) n = 34	(c) n = 50

3) B(700, 0.05) Check--> passes (700)(0.05) ≥ 10 ✓ (700)(0.95)

P(X>50) = 0.0046

N(35, 5.7663)

4) B(400, 0.48) Check --> passes (400)(0.48) ≥ 10 ✓ (400)(0.52)

P(180 < X < 220) = 0.8826

N(192, 9.992)

10) B(2000, 0.5) check --> passes $(2000)(0.5) \geq 10$

$(2000)(0.5)$

P(975 < X < 1050) = 0.8556

N(1000, 22.361)

On the worksheet:
#5, 6, 9

5) B(100, 0.70)

Check:
$100*0.70 \geq 10$
$100*0.30$

N(70, 4.5826)

P(X > 80) = 0.0145

6) B(1500,0.56)

Check:
$1500*0.56 \geq 10$
$1500*0.44$

N(840, 19.225)

P(X > 750) = 0.9999

9) B(1000, 0.10)

Check:
$1000*0.90 \geq 10$
$1000*0.10$

N(100, 9.4868)

P(X < 100) = 50%

p. 402 #18, 29

p. 402

#18) **B(6, 0.80)**

(a) $P(B^c \cap B^c \cap B) = (0.20)(0.20)(0.80) =$ ***0.032***

(b) B(6, 0.20) p = 0.20 because we are concerned with MISSING
$P(X \geq 1) = 1 - P(X \leq 0) =$ binomcdf(6, 0.20, 0) = ***0.7379***

(c) $P(B^c \cap B^c \cap B^c \cap B) = 0.0064$
$P(B^c \cap B^c \cap B^c \cap B^c \cap B) = 0.00128$ = ***0.00768***

(d) P(X = 4) = binompdf(6, 0.80, 4) = ***0.24576***

(e) $P(X \geq 4) = 1 - P(X \leq 3) = 1 -$ binomcdf(6, 0.80, 3) = ***0.90112***

(f) $P(X \leq 4) =$ binomcdf(6, 0.80, 4) = ***0.34464***

29) B(300, 0.06)

(a) check: (300)(0.06)
(300)(0.94) $\geq$ 10 $\rightarrow$ **N(18, 4.113)**

(b) P(X < 12) = normalcdf(-E99, 12, 18, 4.113) = **0.0723**

(c) P(X > 50) = normalcdf(50, E99, 18, 4.113) = **3.654×10^{-15}**
No, it is not likely.

Book problems:

p. 402 #17 & 21, ~~26~~ & 30

17) B(5, 0.13)

(a) $P(L^c \cap L^c \cap L^c \cap L^c \cap L)$ = (0.87) (0.87) (0.87) (0.87)(0.13) = ***0.07448***

(b) $P(X \geq 1) = 1 - P(X \leq 0)$ = 1 – binomcdf(5, 0.13, 0) = ***0.5016***

(c) second: $P(L^c \cap L) = 0.1131$
third: $P(L^c \cap L^c \cap L) = 0.0984$ = ***0.2115***

(d) P(X = 3) = binompdf(5, 0.13, 3) = ***0.0166***

(e) $P(X \geq 3) = 1 - P(X \leq 2)$ = 1 – binomcdf(5, 0.13, 2) = ***0.0179***

(f) $P(X \leq 3)$ = binomcdf(5, 0.13, 3) = ***0.9987***

21) B(12, 0.87)

(a) μ_X = 12*0.87 = ***10.44 people***
$\sigma_X = \sqrt{12 \cdot 0.87 \cdot 0.13}$ = **1.165 people**

(b) (i) 1 - P(X = 12) = 1 – binompdf(12, 0.87, 12) = ***0.8196***

(ii) $P(X \leq 10)$ = binomcdf(12, 0.87, 10) = ***0.4748***

(iii) P(X = 6) = binompdf(12, 0.87, 6) = ***0.0019***

(iv) $P(X > 6) = 1 - P(X \leq 6)$ = 1 – binomcdf(12, 0.87, 6) = ***0.9978***

26) B(12, 0.125)

(a) P(X = 0) = binompdf(12, 0.125, 0) = **0.2014**

(b) $P(X \geq 2) = 1 - P(X \leq 1)$ = 1 – binomcdf(12, 0.125, 1) = **0.4533**

(c) P(X = 3) + P(X = 4) =
= binompdf(12, 0.125, 3) + binompdf(12, 0.125, 4) = **0.1707**

(d) $P(X \leq 4)$ = binomcdf(12, 0.125, 4) = **0.9887**

30) B(150, 0.125)

(a) μ_X = 18.75 frogs
σ_X = 4.05 frogs

(b) check: 150*0.125 $\geq$ 10 N(18.75, 4.05)
150*0.875

(c) P(X > 22) = 0.211

No

Ch. 17 Classwork